

Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

$$1) \frac{2x}{(x+3)(3x+1)} \quad \boxed{\frac{A}{x+3} + \frac{B}{3x+1}}$$

$$2) \frac{1}{x^3+2x^2+x} \quad \boxed{\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}}$$

$$3) \frac{2}{x^2+3x-4} \quad \boxed{\frac{A}{x+4} + \frac{B}{x-1}}$$

$$4) \frac{x^2}{(x-1)(x^2+x+1)} \quad \boxed{\frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}}$$

$$5) \frac{x^3}{x^2+4x+3} \quad \boxed{x-4 + \frac{A}{x+1} + \frac{B}{x+3}}$$

$$6) \frac{2x+1}{(x+1)^3(x^2+4)^2} \quad \boxed{\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}}$$

$$7) \frac{x^4}{x^4-1} \quad \boxed{1 + \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}}$$

$$8) \frac{t^4+t^2+1}{(t^2+1)(t^2+4)^2} \quad \boxed{\frac{At+B}{t^2+1} + \frac{Ct+D}{t^2+4} + \frac{Et+F}{(t^2+4)^2}}$$

Evaluate the integral

$$9) \int \frac{x}{x-6} dx \quad \boxed{x + 6 \ln|x-6| + C}$$

$$10) \int \frac{r^2}{r+4} dr \quad \boxed{\frac{1}{2}r^2 - 4r + 16 \ln|r+4| + C}$$

$$11) \int \frac{x-9}{(x+5)(x-2)} dx \quad \boxed{2 \ln|x+5| - \ln|x-2| + C}$$

$$12) \int_2^3 \frac{1}{x^2-1} dx \quad \boxed{\frac{1}{2} \ln\left(\frac{3}{2}\right)}$$

$$13) \int_0^1 \frac{2x+3}{(x+1)^2} dx \quad \boxed{2 \ln 2 + \frac{1}{2}}$$

$$14) \int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6} dx \quad \boxed{\frac{3}{2} + \ln\left(\frac{3}{2}\right)}$$

$$15) \int \frac{x^2}{(x+1)^3} dx \quad \boxed{\ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C}$$

$$16) \int \frac{x^4 + 1}{x(x^2 + 1)^2} dx \quad \boxed{\ln|x| + \frac{1}{x^2 + 1} + C}$$

Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$17) \int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx \quad [\text{Hint: substitute } u = \sqrt[6]{x}] \quad \boxed{2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C}$$

$$18) \int \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx \quad \boxed{\ln\left[\frac{(e^x + 2)^2}{e^x + 1}\right] + C}$$